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CONSTANT HEAD PERMEAMETER FORMULA DEPENDENCE ON ALPHA PARAMETER

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ABSTRACT. *Reliable estimates of saturated hydraulic conductivity are prerequisites for accurate estimations of water flow and chemical transport through soil profiles. One useful method for evaluating the saturated hydraulic conductivity in the field (Kfs) is the constant-head well permeameter (CHWP) method. The governing equation for this test is based upon flow of water from a cylindrical hole and flow theory for unsaturated-saturated steady flow from the hole. By making assumptions concerning the flow from the well, a solution to the equation is derived which includes coefficients obtained from measurements taken during the test and two parameters related to the soil type being studied. An important parameter in this solution method is the α^* parameter, the ratio of Kfs to the matric flux potential. Relationships are derived and presented which describe the sensitivity of the CHWP solution to variations in α^* . Results show the solution for Kfs is most sensitive to error in the estimate of α^* when $\alpha^* < 0.015 \text{ mm}^{-1}$, corresponding to fine-textured and compacted clay soils. The error in Kfs introduced by poor estimates of α^* is shown to be as large as 210%, for some cases. Thus, the current classification system used to evaluate α^* for this test appears to be inadequate to produce reasonable estimates in Kfs. **Keywords.** Hydraulic conductivity, Infiltration, Parameter estimation.*

The constant-head well permeameter (CHWP) technique (Amoozegar, 1989a; Amoozegar and Warrick, 1986) is a useful technique for evaluating field measurements of saturated hydraulic conductivity (Kfs). The method is frequently used to characterize Kfs (Zangar, 1953; Reynolds et al., 1985, 1986; Reynolds and Elrick, 1985, 1986; Amoozegar and Warrick, 1986; Stephens et al., 1987; Elrick et al., 1989; Amoozegar, 1989a; Elrick and Reynolds, 1992; Xiang, 1994). The constant-head permeameter is designed to maintain a constant head of water at the bottom of a borehole within the unsaturated zone. The flow rate from the device is measured and used to calculate Kfs for the known head of water.

Measurements of Kfs by the CHWP method are obtained under conditions of saturated-unsaturated, three-dimensional flow into unsaturated porous media. Following Reynolds et al. (1985), an equation for steady-state flow, Q ($\text{L}^3 \text{T}^{-1}$), from an auger hole into a uniform unsaturated soil can be derived.

$$Q = \frac{Kfs}{C} \left(2\pi H^2 + C\pi r^2 + 2\pi \frac{H}{\alpha^*} \right) \quad (1)$$

Kfs (L T^{-1}) is the field-saturated hydraulic conductivity, α^* (L^{-1}) is the ratio of Kfs to the matric flux potential (Elrick et al., 1989), H (L) is the constant height of ponded water in the well, r (L) is the radius of the well, and C is a dimensionless shape factor. Equivalently, Kfs can be calculated.

$$Kfs = \frac{CQ}{2\pi H^2 + C\pi r^2 + 2\pi \frac{H}{\alpha^*}} \quad (2)$$

The primary differences in the solution forms of equation 2 are in the components assumed negligible and in the methods used to determine C and α^* . The parameters Q , H , and r are defined by the test. The C factor is a function of the initial soil-water content, soil type, measurement depth below the ground surface, H , and r . This factor can be estimated from numerically derived values based upon numerical solution of Richards' equation for unsaturated flow from an uncased well (Reynolds and Elrick, 1987). For a wide variety of soils and typical H to r ratios, C ranges from 1.25 to 2.25.

Complete solution of equation 2 for Kfs also requires estimating α^* . This solution requires either two CHWP tests, resulting in two equations with two unknowns (Kfs and α^*), or prior estimates of α^* . If two concurrent CHWP tests are made, the simultaneous equations approach (Elrick et al., 1989) can be used to solve for Kfs and α^* . However, in cases where the soil is heterogeneous this approach can result in calculations of negative values of Kfs (Elrick et al., 1989; Amoozegar, 1989b). When this is encountered, Elrick et al. (1989) recommend carrying out two single-height calculations of Kfs using estimates of α^* based upon soil texture.

For cases where only one measurement of flow rate per site is made, Elrick and Reynolds (1992) presented a table

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Table 1. Suggested values for α^* based on soil structural/textural considerations (Elrick and Reynolds, 1992)

α^* (mm ⁻¹)	Comments	Hydrologic Soil Group
∞	Laplace solution (Reynolds and Elrick, 1985). Coarse material	A
0.036	Coarse sands and highly structured soils	A
0.012	Most structured soils and medium and fine sands	B
0.004	Unstructured fine-textured soils.	C
0.001	Compacted clays (e.g., clay liners)	D
0	Gardner solution (Reynolds and Elrick, 1985)	D

of α^* values appropriate for different porous media (table 1). To quantify the error associated with improper selection of the α^* parameter, an analysis of the sensitivity of the Kfs solution to variations in α^* was conducted.

MATHEMATICAL MODEL

A graphical examination of Kfs as a function of α^* indicates Kfs is relatively insensitive to changes in α^* for $\alpha^* > 0.015$ (mm⁻¹) (fig. 1). This corresponds to the range of α^* appropriate for most coarse textured soils (table 1). While the value of Kfs changes for different soils and flow conditions, the relative shape of the Kfs curve (fig. 1) remains the same. This is due to the form of the Kfs function (eq. 2). If Kfs is divided by any maximum Kfs value for a range of α^* , it is apparent that the relative change in Kfs is only a function of H, r, and α^* .

The sensitivity of Kfs with respect to α^* can be better examined by differentiating equation 2 with respect to α^* for constant C, Q, H, and r.

$$\frac{d Kfs}{d \alpha^*} = \frac{2 \pi C H Q}{\alpha^{*2} \left(2 \pi H^2 + C \pi r^2 + 2 \pi \frac{H}{\alpha^*} \right)^2} \quad (3)$$

These conditions would exist for any CHWP field test for Kfs. As equation 3 indicates, the sensitivity of Kfs to changes in α^* is a function of C, Q, H, r, and α^* . The greatest absolute changes in Kfs as a response to errors in estimating α^* occur for high Q, low α^* , conditions. Fortunately, as shown in table 1, low α^* is associated with fine-textured soils where low flow rates are expected. The relative rate of change, defined here as the derivative divided by Kfs, is not a function of Q, only of H, r, α^* , and indirectly C. Thus, the relative rate of change of Kfs with respect to α^* will remain the same regardless of the flow rate. The sensitivity of Kfs to changes in α^* increases as H

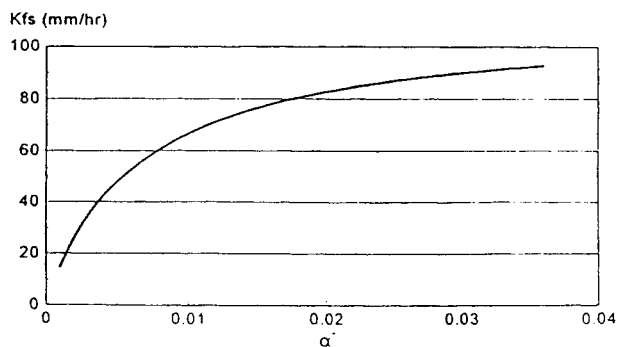


Figure 1—Kfs as a function of α^* for C = 1.6, H = 150 mm, r = 32 mm, and Q = 1×10^7 mm³ h⁻¹.

decreases. However, on a relative basis, the value of H does not have a large influence on the sensitivity of the Kfs solution to errors in α^* . For all cases, the greatest sensitivity is in the range where $\alpha^* < 0.015$ mm⁻¹.

APPLICATION

The derivative, equation 3, as a function of variable α^* , is illustrated in figure 2 for typical values of C, Q, H, and r. The greatest sensitivity to inaccurate estimates in α^* , occurs at the lower end of the α^* scale. This is true for all practical combinations of C, Q, H, and r. The lower end of the α^* scale corresponds to clays and fine-textured soils (table 1). For coarser-textured soils ($\alpha^* > 0.015$ mm⁻¹) the solution is less sensitive to variations in α^* . From a practical standpoint, the most subjectivity will be introduced when the soils being analyzed are fine to medium textured, α^* from 0.004 to 0.012 mm⁻¹. In this range, a 0.7 to 0.3% change will occur in Kfs for each 1% change in α^* (fig. 3). A selection of α^* for a medium to fine sand (0.012 mm⁻¹) rather than for an unstructured fine-textured soil (0.004 mm⁻¹) would constitute a 200% error and could result in a rather significant 140% error in the Kfs estimate.

Small differences in the derivative are also observed as H is varied (fig. 3). In general, the greater the head, the less responsive the formula to changes in α^* . This was also observed by Elrick et al. (1989). To decrease the errors associated with a poor choice in α^* , a large head of water in the CHWP hole should be used. Prior research (Reynolds et al., 1983) has established that the best H to r

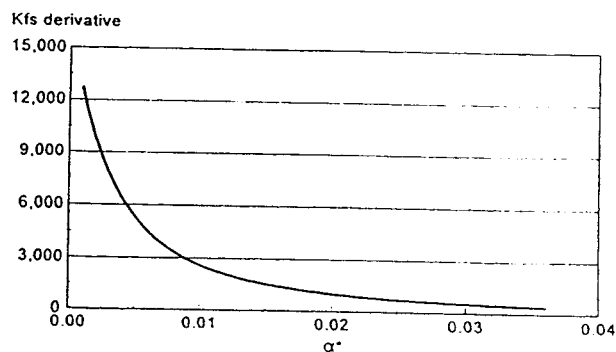


Figure 2—Kfs derivative as a function of α^* for C = 1.6, H = 150 mm, r = 32 mm, and Q = 1×10^7 mm³ h⁻¹.

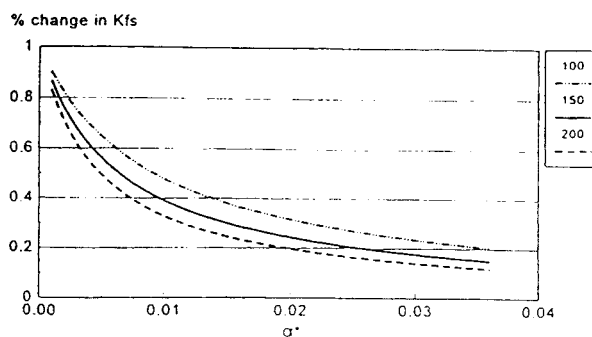


Figure 3—Percent change in Kfs as a response to 1% change in α^* for C = 1.6, r = 32 mm, Q = 1×10^7 mm³ h⁻¹, and H = 100, 150, and 200 mm.

ratio should be approximately 5. By maintaining an H to r ratio greater than 5, the three-dimensional pressure flow component of the flux is maximized and the one-dimensional gravitational flux component is minimized. This conforms to the constraints of the CHWP method. A H to r ratio greater than 5 appears to be adequate for decreasing the sensitivity of the solution to errors in the selection of α^* . Increasing the water depth, H, could help to minimize the effects of improper α^* selection. However, if an estimate of the matric flux potential is desired, increasing H increases the sensitivity of the potential to errors in α^* .

SUMMARY

The current guidelines for estimating α^* are rather broad (table 1). Errors in selecting α^* could be as large as 200 to 300% if an improper classification were selected. This could result in errors in the estimation of Kfs from 40 to 240%. Observed variances in Kfs are commonly as large as 100% (Jury, 1985). Thus, the error introduced by a poor selection in α^* could have a dramatic effect on the overall accuracy of the Kfs estimate. The current classification system used to evaluate α^* for this test appears to be inadequate to produce reasonable estimates of Kfs. Because of the increased sensitivity of Kfs to changes in α^* for $\alpha^* < 0.015 \text{ mm}^{-1}$, one may wish to conduct two measurements and solve for Kfs and α^* using the simultaneous equations method when taking measurements in fine-textured and compacted clay soils. However, it has been shown (Elrick et al., 1989; Amoozegar, 1989b) that in cases where the soil is heterogeneous the simultaneous equations approach can result in calculations of negative values of Kfs. When this is encountered, Elrick et al. (1989) recommend carrying out two single-height calculations of Kfs using estimates of α^* based upon soil texture. A recommendation which may be difficult to follow based upon the results of this analysis.

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